

Over the last several years, many scientists have urged the world around us to unravel the complex web of interconnections that characterize seemingly diverse social [1], biological [2, 3] and technological systems [4, 5]. These systems have been shown to exhibit common features that can be captured using the tools of complex systems theory or in more recent terms, network modeling. At the same time, network models of diverse kinds have been proposed with the aim of describing and explaining the properties of real webs [6, 7]. It turns out that most real networks are better described by growing models in which the number of nodes (or elements) forming the network increases with time and that the probability that a given node has k connections to other nodes follows a power-law distribution $\sim k^{-\gamma}$, with $\gamma \leq 3$. Additionally, the study of complex systems taking place on top of these networks has led to the reconsideration of classical results obtained for regular lattices and random graphs due to the radical changes of the network dynamics when the heterogeneity of complex systems can not be neglected [8, 9, 10, 11].

The first scale-free network model, introduced by Barabási and Albert (BA), postulated that there are two essential ingredients of many real networks [12, 13]: a growing character and the preferential attachment mechanism. The first ingredient states that the network grows

with arbitrary γ -exponents, and non-random correlations can be found nowadays in the scientific literature. On the other hand, there are some models in which the neighborhood is limited to a neighborhood due to geographic constraints [14], or where its linear character is investigated recently, Caldarelli *et al.* [16] have shown that it is possible to produce SF networks without assuming preferential attachment at all. As a byproduct, other properties of the network fit well with those of real-world graphs. Watts introduced an intrinsic fitness model in which the probability of being connected with a probability that depends on the node's fitness. Note, additionally, that the way in which the fitness parameter was introduced is different from the model in [17].

In this paper, we adopt a different perspective. Our aim is to test to what extent the global character of the network in the PA rule in the original BA model is important. We introduce a model in which the PA is applied only to the neighborhood of the newly added node depending on the value of a variable which measures the affinity between the new node and the existing nodes. By going down from the BA limit of the model to the limit where all nodes are distinct, we investigate to what extent the global knowledge of each node's fitness is fundamental to get a scale-free graph.

$$\Pi(k_i) = \frac{k_i}{\sum_{s \in A} k_s} \quad (1)$$

ally v) Repeat steps *ii-iv* such that the final size of the network is $N = m_o + t$.

After t time steps a network made up of N nodes is produced. It is worth mentioning that the inclusion of the parameter a is not a mere artifact. Indeed, most networks are formed by non-identical elements and it is natural to assume that although a given node may have a large connectivity a newly created element will not link to that node because they have very little information. This feature is clearly manifested in social networks like the WWW –where individuals bookmark web pages accordingly to their “affinity”– or in citation networks [19]. In this way, it is very difficult to find a citation in a condensed matter paper that refers to a paper wrote by a psychologist. Additionally, the same argument can be translated to biological networks such as predator-prey webs or protein-protein interaction networks.

Thus, when μ is large enough as to dilute the first effect of the model, we recover the BA model. The problem consists of determining to what extend the logarithmic attachment will give the same results, or in other words, does the knowledge of the entire network really contribute to the properties observed in the

FIG. 1: Number of nodes with connectivity k for different values of μ . The size of the network is $N = 10^4$ and $m_o = m = 3$. The power-law distribution has an exponent equal to 3. Note that the BA limit corresponds to

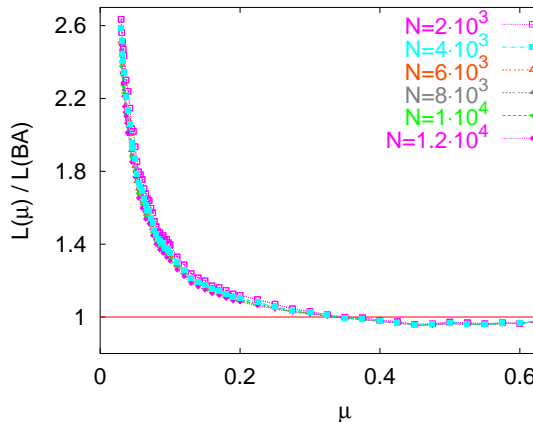


FIG. 2: Ratio between the average shortest path length for different μ values, $L(\mu)$, and that of the BA network, $L(\text{BA})$, for several system sizes. The horizontal line marks the value 1. A transition from graphs fulfilling the small-world properties to a regime in which networks break down in many small clusters, increasing the value of $L(\mu)$ is observed. See the text for details.

work is made up of $N = 500$ nodes.

es, but there is nothing that guarantees *a priori* components of the network will link together in a way that other properties will not be affected. the case of the average shortest path length L . average shortest path length of a graph is defined as m number of nodes one has to pass by to go node of the network to another randomly chosen averaged over all possible pairs of nodes. Complex s show the noticeable property, known as small-property, that the average path length increases h the logarithm of its size. We expect that for es of μ the network is composed by a unique mponent and no fragmentation arises. When e to which the affinity criterion is applied de-the network will gradually loose its compactness stretch approaching a one-dimensional structure e small components. Further reduction of μ pro-e break down of the network in many isolated

es 2 and 3 substantiate this picture. Figure 2 ts the ratio between the average path length ob- or different values of μ and that of the BA net- or several system sizes. As μ restricts the PA ne network undergoes a transition characterized

FIG. 4: Average clustering coefficient c_k of nodes with k for five different values of the parameter μ . No μ decreases, the clustering coefficient departs from limit ($\mu = 1$). The parameters used for the generated networks are as of fig. 1.

efficient c_i . The clustering coefficient of a node defined as the ratio between the number of edges the k_i neighbors of i and its maximum possible $k_i(k_i - 1)/2$, i.e., $c_i = \frac{2e_i}{k_i(k_i - 1)}$. In this way, the clustering coefficient, c is given by the average all nodes of the network. The clustering coefficient local character as it gives the probability that with a common neighbor are also linked together it is expected that this magnitude, in our model on the affinity of each node and the range of pre-attachment given by μ . Figure 4 shows the average clustering coefficient of nodes with a given connection for different values of the parameter μ . The exhibits almost no correlations with the degree of vertices and the smallest value for the clustering coefficient. As μ is reduced, the first selection of nodes affinity values plays a more dominant role con- to the rising of c_i for small and large connectivity the transition, $\mu \sim 0.04$, the average coefficient one order of magnitude greater than that of the work

Average nearest neighbor connectivity k_{nn} against k for different values of μ . Results are averaged over 100 network realizations for each μ value. Other parameters are as of fig. 1.

the tendency that networks generated with small values of μ display disassortative mixing at both ends of the connectivity range.

In this paper, we have studied a version of the Barabási-Albert scale-free model that allows to tune the range in which the preferential attachment is applied. The model considers that all nodes are different such that they are in principle unable to link to very distinct nodes. By introducing an affinity selection before applying the preferential attachment rule, we tested whether or not the degree of the entire network is an essential requisite for scale-free networks. Our results seem to support the idea that having at least some degree of preferential

attachment where non-trivial properties arise. In this sense, it would be interesting to perform the same analysis in more realistic growing network models looking for more similarities with real-world networks. For example, the exponential connectivity distribution can be tuned to smoothness by incorporating the first level of selection of the model in the generalized BA model [6], which would allow to give arbitrary γ values in the interval $(2, 3)$.

Acknowledgments

The authors thank F. Falo, J. L. García-Palacios, M. Floría and A. F. Pacheco for helpful comments and discussions. J. G-G acknowledges financial support from CSIC through an I3P-BPD2002-1 grant. Y. Moreno is supported by the Secretaría de Estado de Educación y Universidades (Spain, SB2000-0357). This work was partially supported by the Spanish DGICYT (BFM2002-01798).

[1] E. J. Newman, Proc. Natl. Acad. Sci. U.S.A. **98**, 404 (2001).
 [2] H. Jeong, S. P. Mason, A.-L. Barabási, and Z. N. Oltvai, Nature (London) **411**, 41 (2001).
 [3] F. Solé, and J. M. Montoya, Proc. R. Soc. London B **268**, 2039 (2001).

[11] Y. Moreno, J. B. Gómez, and A. F. Pacheco, Phys. Rev. E **68**, 035103(R) (2003).
 [12] A.-L. Barabási, and R. Albert, Science **286**, 509 (1999).
 [13] A.-L. Barabási, R. Albert, and H. Jeong, Phys. Rev. E **60**, 173 (1999).
 [14] S. Mossa, M. Barthelemy, H. E. Stanley, and A. F. Pacheco, Phys. Rev. E **68**, 035103(R) (2003).